Resonant absorption at the vortex-core states in d-wave superconductors

N. B. Kopnin

L.D. Landau Institute for Theoretical Physics, 117334 Moscow, Russia;
Low temperature Laboratory, Helsinki University of Technology
P.O. Box 2200, FIN-02015 HUT, Finland
(February 1, 2008)

We predict a resonant microwave absorption on collective vortex modes in a superclean d-wave superconductor at low temperatures. Energies of the collective modes are multiples of the distance between the exact quantum levels of bound states in the vortex core at lower temperatures and involve delocalized states for higher temperatures. The characteristic resonant frequency is larger than the cyclotron frequency ω_c but lower than the Caroli-deGennes-Matricon minigap Δ^2/E_F; it has a $\sqrt{H}$ dependence on the magnetic field and decreases down to ω_c with increasing temperature. We calculate the vortex mass as a response to a slow acceleration. This mass is equal, by the order of magnitude, to the mass of electrons inside the vortex "core" with dimensions ξ by $\xi\sqrt{H_{c2}/H}$; it increases with temperature. We discuss the universal flux-flow regime predicted in[1] and show that it exists in a broader temperature range than it has been originally found.

I. INTRODUCTION

The vortex dynamics in clean superconductors is determined by localized states in vortex cores. Motion of a vortex excites transitions between the states; a competition between the relaxation rate $1/\tau$ and the interlevel spacing controls the proportion between dissipative and reactive forces experienced by the vortex[2,3]. In d-wave superconductors, the vortex dynamics is expected to be more intricate due to a peculiar structure of the vortex-core states. The presence of gap nodes introduces the most important difference in the structure of core states compared to an s-wave superconductor. As was shown in Refs.[1,4], instead of a well-defined quasiclassical Caroli-deGennes-Matricon[5] interlevel spacing ω_0, the true quantum states in d-wave superconductors have a much smaller separation between quantum levels, E_0, which depends on the magnetic field. As a result, there appears another parameter $E_0\tau$ which influences the vortex dynamics. According to Ref.[1], a new universal regime can be reached in superclean superconductors with longitudinal and transverse components of the conductivity tensor independent of the relaxation time. It is realized when the relaxation rate is smaller than the average distance between the quasiclassical energy levels $\langle\omega_0\rangle \sim T_c^2/E_F$ but larger than the separation between the true quantum-mechanical states E_0.

In the present paper, we study both steady and oscillatory motion of vortices at low temperatures $T \ll T_c$ and low magnetic fields $H \ll H_{c2}$ in more detail using the microscopic theory outlined in Ref.[1]. For a steady flux flow, we find that the universal regime is realized in a considerably broader region of temperatures and magnetic fields than what was originally predicted in Ref.[1]. This conclusion agrees with the results of Ref.[6]. The relevant parameter which governs the vortex dynamics is shown to be the energy of "collective modes" $E_{\rm col}$. We calculate $E_{\rm col}$ and show that it coincides with the true quantum mechanical interlevel spacing E_0 if the energy of excitations is $\epsilon \ll T_c\sqrt{H/H_{c2}}$, and involves delocalized states for excitations with higher energies $\epsilon \gg T_c\sqrt{H/H_{c2}}$. For an oscillatory motion of vortices which can be excited by a microwave irradiation, we predict a finite resonant absorption even for infinitely long relaxation time. The series of resonances occurs at multiples of $E_{\rm col}$ with the resonant frequencies considerably lower than the average quasiclassical interlevel distance T_c^2/E_F but higher than the cyclotron frequency ω_c. They have a $\sqrt{H}$ magnetic field dependence at very low temperatures. For higher temperatures, the series of resonances compresses down to an absorption edge which approaches ω_c for $T \sim T_c$.

II. NONEQUILIBRIUM EXCITATIONS

A. Spectrum

We consider superconductors which have the coherence length ξ much longer than the inverse Fermi momentum, $p_F\xi \gg 1$. For these superconductors, a quasiclassical approximation is appropriate. A quasiclassical particle moves along a straight line parallel to its momentum which is conserved with the accuracy $(p_F\xi)^{-1}$. The angular momentum is thus another approximately conserved quantity even if the order parameter does not have a cylindrical symmetry; it is now a continuous variable not directly related with labelling the quantum states in the vortex core. A quasiparticle passing at some distance near a vortex can become classically localized in the vortex core region. It will have an energy characterized by the momentum along the vortex axis p_z, the momentum direction in the plane perpendicular to the vortex axis,

and by the impact parameter $b = -\mu/p_\perp$ coupled to the angular momentum μ.

For a d-wave superconductor without admixture of other components, the order parameter is

$$\Delta_{\mathbf{p}} = \Delta_{0\mathbf{p}} e^{i\phi} = \Delta_0 \sin(2\alpha) e^{i\phi} \qquad (1)$$

where α is the angle between $\mathbf{p}_\perp$ and the x axis which is taken along one of the gap nodes. The modulus of the order parameter $\Delta_0 = \Delta_0(\rho, \phi)$; at large distances, $\Delta_0 = \Delta_\infty$. We take the z axis along the vortex circulation, $\mathbf{z} = \mathrm{sign}\,(e)\mathbf{H}/H$, and denote by $\mathbf{v}_\perp$ the quasiparticle velocity in the (x, y) plane. If ρ and ϕ are the distance and the azimuthal angle in the cylindrical frame, the impact parameter b of a quasiparticle moving through the point (ρ, ϕ) and the distance s along the trajectory are

$$b = \rho \sin(\phi - \alpha); \quad s = \rho \cos(\phi - \alpha). \qquad (2)$$

The quasiparticle energy can be found using the quasiclassical Green function technique in the same way as it has been done for s-wave superconductors in Ref.[7]. Here we summarize the most important properties of the energy spectrum of a quasiclassical particle in the vortex core. It is discussed in more detail in Appendix. The quasiclassical spectrum has several branches belonging to various values of the radial quantum number n. The anomalous branch with $n = 0$ crosses zero of energy as a function of the impact parameter. For small impact parameters $b \ll \xi/|\sin(2\alpha)|$,

$$\epsilon_0(\alpha, b) = \frac{2\Delta_\infty^2 \sin^2(2\alpha) L b}{v_\perp} + \frac{p_\perp b \omega_c}{2} \qquad (3)$$

where $L = \ln[1/|\sin(2\alpha)|]$ for $b \ll \xi$ and $L = \ln[\xi v_\perp/b v_F |\sin(2\alpha)|]$ for $\xi \ll b \ll \xi/|\sin(2\alpha)|$; the second term is the magnetic energy where $\omega_c = |e|H/m_\perp c$ is the cyclotron frequency with $m_\perp = p_\perp/v_\perp$. Modulus of charge appears due to the choice of the z axis. Note that the magnetic energy has the energy level spacing[8] equal to the Larmor frequency $\omega_c/2$ rather than just to ω_c as it would be the case for the Landau spectrum in the normal state. This is because the wave function is localized at distances of the general order of ξ rather than at the magnetic radius $\rho_H \sim p_\perp c/eH$ which would be the case in the normal state. Near the gap nodes, i.e., when $\sin(2\alpha)$ is small, the low-lying states with energies much below the gap at infinity, $\epsilon \ll \Delta_0|\sin 2\alpha|$, correspond to $b \ll \xi/|\sin 2\alpha|$. The derivative of the energy with respect to the angular momentum, $\omega_0(\alpha) = p_\perp^{-1}[\partial\epsilon_0(\alpha, b)/\partial b]$, is the analogue of the Caroli-deGennes-Matricon minigap[5] in an s-wave superconductor.

For large impact parameters, $b \gg \xi/|\sin 2\alpha|$, the energy is

$$\epsilon_0(b) = \mathrm{sign}\,(b)\left(\Delta_\infty |\sin(2\alpha)| - \frac{v_\perp}{4|b|}\right) + \frac{p_\perp b \omega_c}{2}. \qquad (4)$$

The particles are localized on lines passing through the vortex perpendicular to the momentum direction.

The localization length along the trajectory is $s \sim \sqrt{\xi b/|\sin(2\alpha)|} \ll b$. A similar spectrum was considered in Ref.[6].

Eqs. (3) and (4) can be combined into a single interpolation formula

$$\epsilon_0(b) = \frac{\Delta_\infty^2 \sin^2(2\alpha) b}{|b|\Delta_\infty| \sin(2\alpha)| + \beta v_\perp} + \frac{p_\perp b \omega_c}{2} \qquad (5)$$

where β is a constant. The fit to the large-b region of energy gives $\beta = 1/4$; the best fit for low-b region is reached with $\beta = 1/(2L)$. We define an angle α_0 at which the particle energy $\tilde\epsilon = \epsilon_0(b) - p_\perp b \omega_c/2$ is equal to the gap $\tilde\epsilon = \Delta_\infty|\sin 2\alpha_0|$; the angle α_0 is close to one of the nodes if $\tilde\epsilon \ll \Delta_\infty$. It is a quasiparticle moving at an angle $\alpha > \alpha_0$ which is classically localized.

Other branches with nonzero radial quantum numbers n are separated from $\epsilon_0(b)$ by energies of the order of $\Delta_\infty|\sin(2\alpha)|$. The number of branches with energies between $\tilde\epsilon$ and $\Delta_\infty|\sin(2\alpha)|$ is

$$\nu \sim \frac{\Delta_\infty|\sin(2\alpha)|}{\sqrt{[\Delta_\infty^2 \sin^2(2\alpha) - \tilde\epsilon^2]}}. \qquad (6)$$

Eqs. (4) and (5) are quantitatively correct as long as $b \ll \rho_{\max}$ where $\rho_{\max} \sim \xi\sqrt{H_{c2}/H}$ is the distance between vortices. Such distances are, in principle, accessible for a particle which moves near the direction of a gap node within the angle $\alpha \sim \sqrt{H/H_{c2}}$. We shall see, however, that the condition $b \ll \rho_{\max}$ is fulfilled for energies $\epsilon \ll \Delta_\infty \sqrt{H/H_{c2}}$.

A classically localized particle is not necessarily localized in a strict quantum-mechanical sense. This was also found in numerical calculations of Ref.[9]. The true quantum levels can be obtained from the semiclassical Bohr-Sommerfeld quantization rule[1,10,11]

$$\oint \mu(\alpha)\, d\alpha = 2\pi\left(m + \frac{1}{2}\right); \qquad (7)$$

m is an integer, $\frac{1}{2}$ appears because the single-particle wave function changes its sign after encircling a single-quantum vortex. The angular momentum $\mu(\alpha)$ is expressed through the quasiparticle energy according to Eq. (5). Consider first an energy $\epsilon \ll \Delta_\infty \sqrt{H/H_{c2}}$. The integral in Eq. (7) converges according to Eq. (5) and is determined by angles $\alpha \sim \sqrt{H/H_{c2}}$. For this region of angles, the spectrum is actually given by the exact equation (3): $\epsilon_0(\mu) = -\omega_0(\alpha)\mu$ where

$$\omega_0(\alpha) = 8\Omega_0 \alpha^2 + \omega_c/2 \qquad (8)$$

with

$$\Omega_0 = (\Delta_\infty^2/v_\perp p_\perp) \ln[(\Delta_\infty/\epsilon)\sqrt{H/H_{c2}}]. \qquad (9)$$

The characteristic impact parameters are of the order of $b \sim \epsilon/p_\perp \omega_c$, i.e., $b \ll \rho_{\max}$. We obtain $E_m = E_0(m + 1/2)$ where

$$E_0 = \sqrt{\Omega_0 \omega_c}. \qquad (10)$$

A qualitative expression for the spectrum with the $\sqrt{H}$ dependence on the magnetic field similar to Eq. (10) was found in Ref.[4] neglecting the magnetic energy. The argumentation was that the divergence of $\omega_0^{-1}(\alpha)$ without the magnetic energy should be cut off at such angles α that the characteristic distance of a qusiparticle from the vortex axis is of the order of the intervortex distance $\rho_{\max}$; here the corrections to the quasiparticle energy induced by neighboring vortices is just of the order of $\omega_0(\alpha)$. We see, however, that Eq. (10) is obtained exactly if one takes into account the magnetic quantization. We find simultaneously that the particles with energies $\epsilon \ll \Delta\sqrt{H/H_{c2}}$ are acually localized in the vortex core since $b \ll \rho_{\max}$; thus the corrections from neighboring vortices are not important. For larger energies $\epsilon \sim \Delta_\infty\sqrt{H/H_{c2}}$, the localization radius becomes of the order of $\rho_{\max}$. These particles can not be considered as localized any more because of the presence of other vortices. For higher energies $\epsilon \gg \Delta_\infty\sqrt{H/H_{c2}}$, the part with extended trajectories $\alpha < \alpha_0$ in Eq. (7) dominates, and thus quasiparticles are no longer localized in the core even for an isolated vortex.

The average quasiclassical energy-level spacing $\langle\omega_0\rangle \sim \Omega_0 \sim \Delta_\infty^2/E_F$ determines the parameter $\Omega_0\tau$ which controls the vortex dynamics in clean s-wave superconductors with $\tau \gg T_c^{-1}$. The value $\Omega_0\tau \sim 1$ separates the so called moderately clean limit, $\Omega_0\tau \ll 1$, with a highly dissipative vortex dynamics from the superclean limit, $\Omega_0\tau \gg 1$, where dissipation is small.

In d-wave superconductors, situation is more intriguing. It was predicted in Ref.[1] that a large dissipation persists for much longer ℓ. In the present paper we show that, for a steady vortex motion, the relevant parameter which marks the transition between dissipative and nondissipative dynamics is $E_{\mathrm{col}}\tau$ where E_{col} is the temperature dependent characteristic energy of "collective modes" induced by moving vortex. For low temperatures, it coincides with the energy of "bound states" determined by Eqs. (7), (10): $E_{\mathrm{col}} = E_0 \ll \Omega_0$. The energy E_{col} decreases down to ω_c when the temperature approaches T_c. The parameter $\omega_c\tau$ is much smaller than $\Omega_0\tau$ since $\omega_c \sim (H/H_{c2})\Omega_0$. We have thus a hierarchy of energies $\omega_c \ll E_{\mathrm{col}} \ll \Omega_0$. To get the parameter which controls the vortex dynamics, these energies should be compared either with the relaxation rate $1/\tau$ for a steady motion of vortices or with $\max(1/\tau, \omega)$ where ω is the characteristic frequency of vortex oscillations. Large values of $\Omega_0\tau$ require a very high purity of samples: the mean free path should be $\ell \gg (T_c/E_F)\xi$. Nevertheless, there are experimental evidences that such a regime can be realized in practice.[12]

B. Balance of forces on a moving vortex

The force on a vortex from the environment where it moves is the momentum transferred from the excitations. The exact microscopic expression for the force has been derived in Ref.[3]. It can equivalently be presented as a force produced by quasiclassical particles with a distribution f, characterized by canonically conjugated coordinate α and angular momentum μ[13]. The contribution from classically localized states is

$$\mathbf{F}_{\mathrm{env}}^{(\mathrm{loc})} = -\sum_n \int \frac{dp_z}{2\pi} \frac{d\mu\,d\alpha}{2\pi} \frac{\partial \mathbf{p}_n}{\partial t} f. \qquad (11)$$

Using $\partial \mathbf{p}/\partial t = -\nabla\epsilon_n = [\hat{\mathbf{z}} \times \mathbf{p}](\partial\epsilon_n/\partial\mu)$ we get

$$\mathbf{F}_{\mathrm{env}}^{(\mathrm{loc})} = -\sum_n \int \frac{dp_z}{2\pi} \frac{d\mu\,d\alpha}{2\pi} [\hat{\mathbf{z}} \times \mathbf{p}_\perp] \frac{\partial\epsilon_n}{\partial\mu} \delta f_n$$
$$= \pi N [\hat{\mathbf{z}} \times \mathbf{u}] \left\langle \sum_n \int \gamma_H \frac{df_0}{d\epsilon} \frac{\partial\epsilon_n}{\partial\mu}\, d\mu \right\rangle_F$$
$$- \pi N \mathbf{u} \left\langle \sum_n \int \gamma_O \frac{df_0}{d\epsilon} \frac{\partial\epsilon_n}{\partial\mu}\, d\mu \right\rangle_F. \qquad (12)$$

We take the nonequilibrium distribution function in the form

$$\delta f = -\frac{df_0}{d\epsilon}\left((\mathbf{u}\mathbf{p}_\perp)\gamma_H + ([\mathbf{u} \times \mathbf{p}_\perp]\,\mathbf{z})\,\gamma_O\right) \qquad (13)$$

where f_0 is the equilibrium Fermi distribution, the factors γ_O and γ_H describe the longitudinal and transverse responses to the vortex velocity $\mathbf{u}$, and N is the electron density; $\langle\cdots\rangle_F$ is the average over the whole Fermi surface which includes averaging over α. We assume here a Fermi surface with not less than the tetragonal symmetry in the plane perpendicular to the vortex axis. For simplicity, we consider only the particle-like Fermi surface, the generalization for a surface with both particle-like and hole-like parts is given later.

The force from classically delocalized states is[3]

$$\mathbf{F}_{\mathrm{env}}^{(\mathrm{del})} = \pi N [\hat{\mathbf{z}} \times \mathbf{u}] \left\langle \int_{\mathrm{del}} \gamma_H \frac{df_0}{d\epsilon}\, d\epsilon \right\rangle_F$$
$$- \pi N \mathbf{u} \left\langle \int_{\mathrm{del}} \gamma_O \frac{df_0}{d\epsilon}\, d\epsilon \right\rangle_F. \qquad (14)$$

The force $\mathbf{F}_{\mathrm{env}}$ should be balanced by the Lorentz force $\mathbf{F}_L = \mathrm{sign}(e)\phi_0[\mathbf{j} \times \hat{\mathbf{z}}]/c$ where $\phi_0 = \pi c/|e|$ is the flux quantum. Since the average electric field is $\mathbf{E} = [\mathbf{B} \times \mathbf{u}]/c$, the force balance $\mathbf{F}_L + \mathbf{F}_{\mathrm{env}} = 0$ gives a linear relation between the transport current and the electric field. The proportionality coefficients are the Ohmic and Hall conductivities. The Ohmic conductivity is

$$\sigma_O = \frac{N|e|c}{B} \left\langle \sum_n \int \gamma_O \frac{df_0}{d\epsilon} \frac{\partial\epsilon_n}{\partial\mu}\, d\mu \right\rangle_F$$
$$+ \frac{N|e|c}{B} \left\langle \int_{\mathrm{del}} \gamma_O \frac{df_0}{d\epsilon}\, d\epsilon \right\rangle_F. \qquad (15)$$

For the Hall conductivity σ_H one has the same expression with γ_H instead of γ_O and $|e|$ replaced with e.

In Galilean invariant systems $\mathbf{j} = N_s e \mathbf{v}_s$, and the force balance is sometimes presented in terms of the Magnus force from the superfluid component and the friction plus transverse forces from the normal component

$$\pi N_s[(\mathbf{v}_s - \mathbf{u}) \times \hat{\mathbf{z}}] - D\mathbf{u} - D'[\hat{\mathbf{z}} \times \mathbf{u}] = 0.$$

In such representation, the Magnus force includes the Lorentz force and a part of the transverse force $\mathbf{F}_{\mathrm{env},\perp}$. The constants D and D' are expressed through the conductivities as

$$\sigma_O = \frac{c^2 D}{\phi_0 B}; \ \sigma_H = \mathrm{sign}\,(e)\frac{c^2(\pi N_s - D')}{\phi_0 B}.$$

III. DISTRIBUTION FUNCTION

A. Kinetic equation

The kinetic equation for the distribution function $f(t, \alpha, \mu)$ for a system of fermions characterized by canonically conjugated variables μ and α has the form[13]

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \alpha}\frac{\partial \epsilon_n}{\partial \mu} - \frac{\partial \epsilon_n}{\partial \alpha}\frac{\partial f}{\partial \mu} = -\frac{f - f_0}{\tau_n}. \tag{16}$$

A particle is classically localized if it has an energy $\tilde{\epsilon} < \Delta_\infty$ and moves at an angle $\alpha > \alpha_0$ counted from one of the nodes. For localized quasiparticles, the derivative $\partial \epsilon_n / \partial \mu = -\omega_n(\alpha)$ is the interlevel spacing. The collision integral is written in a τ-approximation. This equation can also be derived microscopically.[3]

If the vortex moves with a velocity $\mathbf{u}$ with respect to the heat bath, the Doppler shift of the energy $\epsilon \rightarrow \epsilon(\mu, \alpha) - \mathbf{pu}$ produces the "driving force" $(\partial \epsilon_n / \partial \alpha) = [\mathbf{u} \times \mathbf{p}_\perp]\hat{\mathbf{z}}$ acting on quasiparticles. The third term in the l.h.s. of kinetic equation (16) contains this force multiplied by $\partial f_0 / \partial \mu = -\omega_n(\alpha)(df_0/d\epsilon)$. We look for a solution in the form $f - f_0 = \delta f$ where $\delta f = \delta f(\epsilon, \alpha)$ is independent of b. Eq. (16) gives

$$\frac{\partial \delta f}{\partial \alpha} - ([\mathbf{u} \times \mathbf{p}_\perp]\hat{\mathbf{z}})\frac{df_0}{d\epsilon} = U(\alpha)\delta f \tag{17}$$

where

$$U_n(\alpha) = \left(-i\omega + \frac{1}{\tau}\right)\omega_n^{-1}(\alpha). \tag{18}$$

Here we assume that the vortex velocity has a form $\mathbf{u} = \mathbf{u}_\omega e^{-i\omega t}$ where $\omega \ll \Delta$.

Introducing the longitudinal and transverse responses to the vortex velocity according to Eq. (13) and taking into account that, for the tetragonal symmetry, the responses do not depend on the direction of the vortex motion with respect to the crystal lattice, one finds two coupled first-order differential equations for $\gamma_O(\alpha)$ and $\gamma_H(\alpha)$:

$$\frac{\partial \gamma_O}{\partial \alpha} - \gamma_H - U(\alpha)\gamma_O + 1 = 0,$$
$$\frac{\partial \gamma_H}{\partial \alpha} + \gamma_O - U(\alpha)\gamma_H = 0. \tag{19}$$

A quasiparticle is delocalized either for energies $\tilde{\epsilon} > \Delta_\infty$ or for angles $\alpha < \alpha_0$. For a homogeneous magnetic field the distribution function of delocalized electrons was shown[3] to satisfy Eq. (17) where ω_n is replaced with $g\omega_c$. Here g is the number of states for a particle with given $\tilde{\epsilon}$, α, and b at large distances from the vortex, i.e., the quasiclassical Green function g:

$$g = \tilde{\epsilon}/\sqrt{\tilde{\epsilon}^2 - \Delta_0^2 \sin^2 2\alpha}.$$

The distribution function thus has the form of Eq. (13) with γ_O and γ_H satisfying Eqs. (19) where now

$$U(\alpha) = \left(-i\omega + \frac{1}{\tau}\right)(\omega_c g)^{-1}. \tag{20}$$

Since γ_O and γ_H obey first-order differential equations they are continuous functions at $\alpha = \alpha_0$. For $\tilde{\epsilon} > \Delta_\infty$, the potential is given by Eq. (20) in the whole region of angles.

B. Static response

Equations (19) can be easily solved. We have $\gamma_H(\alpha) = \mathrm{Re}\,W$; $\gamma_O(\alpha) = \mathrm{Im}\,W$ where

$$W = e^{[i\alpha + F(\alpha)]}\left(C - i\int_0^\alpha e^{-[i\alpha' + F(\alpha')]}\,d\alpha'\right)$$

with

$$F(\alpha) = \int_0^\alpha U(\alpha')\,d\alpha'.$$

In the moderately clean limit, $\Omega_0\tau \ll 1$, the potential $U(\alpha)$ is always large, and we obtain the local solution as in an s-wave superconductor[3]

$$\gamma_O(\alpha) = \omega_0(\alpha)\tau, \ \gamma_H(\alpha) = [\omega_0(\alpha)\tau]^2. \tag{21}$$

In the superclean limit $\Omega_0\tau \ll 1$, the potential $U(\alpha)$ is small almost everywhere except for vicinities of the gap nodes where it becomes large. Consider first energies $\epsilon \ll \Delta_\infty \sqrt{H/H_{c2}}$ and find the distribution function for the anomalous branch $n = 0$ in the region of angles α not specifically close to the gap nodes. It is this branch which is only excited at temperatures $T \ll T_c\sqrt{H/H_{c2}}$. The overall behavior of the distribution function is to the highest extent determined by what happens in a close

vicinity of the gap nodes. It is this region which is responsible for the whole build-up of nonequilibrium distribution of excitations.

For angles larger than $\alpha \sim \sqrt{H/H_{c2}}$ from the nodes one can neglect the potential U. As a result, one has for $\delta\alpha < \alpha < \pi/2 - \delta\alpha$ where $\sqrt{H/H_{c2}} \ll \delta\alpha \ll 1$,

$$\gamma_O(\alpha) = A\cos\alpha + B\sin\alpha;$$
$$\gamma_H(\alpha) = 1 - A\sin\alpha + B\cos\alpha. \qquad (22)$$

The constants A and B can be found by matching with the solution in the vicinity of the node, $\alpha \ll 1$, where $\gamma_{O,H}(\alpha) \propto e^{F(\alpha)}$. This provides the boundary condition $\gamma_{O,H}(+\delta\alpha) = e^{2\lambda}\gamma_{O,H}(-\delta\alpha)$ across the node at $\alpha = 0$. Here $2\lambda = F(+\delta\alpha) - F(-\delta\alpha)$. For such energies, the region of angles $\alpha < \alpha_0$ with delocalized trajectories is not important. The integral for λ converges and is determined by angles $\alpha \sim \sqrt{H/H_{c2}}$. This range of angles sets the width of the transition region near a gap node where the distribution function jumps from its value at $\alpha = -\delta\alpha$ to its value at $\alpha = +\delta\alpha$. We obtain using Eq. (8)

$$\lambda = \tau^{-1}\int_0^\infty \frac{d\alpha}{\omega_0(\alpha)} = \frac{\pi}{4E_0\tau}. \qquad (23)$$

The solution for $\gamma_{O,H}$ should be periodically continued to the rest of angles with the period $\pi/2$ since the response function has the same tetragonal symmetry as the underlying system: $\gamma(\pi/2 - \delta\alpha) = \gamma(-\delta\alpha)$. Together with the above boundary condition, this gives

$$A = \frac{e^\lambda \sinh\lambda}{2\sinh^2\lambda + 1}; \;\; B = \frac{e^{-\lambda}\sinh\lambda}{2\sinh^2\lambda + 1}. \qquad (24)$$

We have after averaging over the azimuthal angle α

$$\langle\gamma_H\rangle = 1 - \frac{4}{\pi}\frac{\tanh^2\lambda}{1 + \tanh^2\lambda}, \;\; \langle\gamma_O\rangle = \frac{4}{\pi}\frac{\tanh\lambda}{1 + \tanh^2\lambda}. \qquad (25)$$

For energies $\epsilon \gg \Delta_\infty\sqrt{H/H_{c2}}$, the region of angles $\alpha < \alpha_0$ with extended trajectories dominates; now it is α_0 which determines the width of the transition region where γ_O and γ_H jump. Compact expressions can be obtained if we assume that $U(\alpha)$ is independent of α for $\alpha < \alpha_0$. For simplicity, we replace $g\omega_c\tau$ with $\omega_c\tau$. This does not change the results qualitatively. We have for $\alpha < \alpha_0$

$$\gamma_O = d_1 + e^{\alpha/\omega_c\tau}\left(C_1\sin\alpha + C_2\cos\alpha\right),$$
$$\gamma_H = d_2 + e^{\alpha/\omega_c\tau}\left(C_1\cos\alpha - C_2\sin\alpha\right) \qquad (26)$$

where

$$d_1 = \frac{\omega_c\tau}{\omega_c^2\tau^2 + 1}; \;\; d_2 = \frac{\omega_c^2\tau^2}{\omega_c^2\tau^2 + 1}.$$

We can use Eq. (22) for angles $\alpha > \alpha_0 + \delta\alpha$. Since the contribution to λ form the region of angles $\alpha \sim \delta\alpha$ is

much smaller than that from the angles $\alpha \sim \alpha_0$, we can extend Eq. (22) to the angles $\alpha = \alpha_0$ and $\alpha = \pi/2 - \alpha_0$. First, we match equations (22) and (26) at $\alpha = \alpha_0$. Another condition is obtained by matching Eqs. (26) taken at $\alpha = -\alpha_0$ with Eq. (22) taken at $\alpha = \pi/2 - \alpha_0$. We obtain four equations for four constants A, B, C_1, and C_2 where now $\lambda = \alpha_0/\omega_c\tau = \epsilon/(2\Delta_\infty\omega_c\tau)$.

If λ is not considerably smaller than unity, we can neglect α_0 in the boundary conditions, and obtain

$$A = (1 - d_2)\frac{e^\lambda\sinh\lambda}{2\sinh^2\lambda + 1} - d_1\frac{e^{-\lambda}\sinh\lambda}{2\sinh^2\lambda + 1};$$
$$B = (1 - d_2)\frac{e^{-\lambda}\sinh\lambda}{2\sinh^2\lambda + 1} + d_1\frac{e^\lambda\sinh\lambda}{2\sinh^2\lambda + 1}.$$

For values of $\lambda \sim 1$ and larger, one automatically has $\omega_c\tau \ll 1$. As a result, both d_1 and d_2 are small, and we recover Eqs. (24) and (25) with the new definition for λ. The coefficients C_1 and C_2 are in this case

$$C_1 = \frac{\cosh\lambda}{2\sinh^2\lambda + 1}; \;\; C_2 = \frac{\sinh\lambda}{2\sinh^2\lambda + 1}.$$

We see that the distribution function for energies $\Delta_\infty\sqrt{H/H_{c2}} \ll \epsilon \ll \Delta_\infty$ is not small even for delocalized trajectories $\alpha < \alpha_0$.

The contribution from the region of angles $\alpha \sim 1$ to Eq. (15) and to the corresponding equation for σ_H decreases together with λ as the parameter $\omega_c\tau$ increases. For $\lambda \ll 1$, the contribution from angles $\alpha \sim \alpha_0$ where the gap $\Delta_\infty|\sin(2\alpha)|$ is of the order of temperature becomes important. In this case, we obtain from Eqs.(22) and (26)

$$A = \lambda(1 - d_2 - d_1) + \alpha_0(1 - d_2 + d_1);$$
$$B = \lambda(1 - d_2 + d_1) - \alpha_0(1 - d_2 - d_1);$$
$$C_1 = 1 - d_2 + \lambda d_1; \;\; C_2 = \lambda(1 - d_2) - d_1.$$

As a result,

$$\langle\gamma_O\rangle = \frac{4}{\pi}\lambda; \;\; \langle\gamma_H\rangle = 1. \qquad (27)$$

We can thus use Eq. (25) within the whole range of λ and combine the two results for λ in different regions of energy into a single interpolation expression

$$\lambda = \frac{\pi}{4E_{\mathrm{col}}\tau} \qquad (28)$$

where E_{col} has the meaning of a characteristic energy of "collective modes"

$$E_{\mathrm{col}} = \left[\frac{1}{\sqrt{\Omega_0\omega_c}} + \frac{2|\epsilon|}{\pi\Delta_\infty\omega_c}\right]^{-1} \qquad (29)$$

which we discuss later.

For energies $\epsilon > \Delta_\infty$ we have from Eqs. (26) $C_1 = C_2 = 0$ and

$$\gamma_O = d_1; \gamma_H = d_2. \qquad (30)$$

C. Frequency-dependent response

Assume that the external frequency $\omega \ll \Omega_0$. In this limit, the solution of the kinetic equation (16) has the form of Eqs. (25,28) where $1/\tau$ in Eq. (28) is replaced with $1/\tau - i\omega$. Consider first the case $\omega \gg \omega_c$. The factors $d_1 \sim \omega_c/\omega$ and $d_2 \sim \omega_c^2/\omega^2$ are small. In the limit $\omega\tau \to \infty$, the longitudinal response is

$$\langle \gamma_O \rangle = i \frac{4}{\pi} \frac{\tan \lambda'}{\tan^2 \lambda' - 1} \tag{31}$$

where

$$\lambda' = \frac{\pi\omega}{4E_{\text{col}}}. \tag{32}$$

The response has poles at $\lambda' = \pi/4 + \pi k/2$ or at frequencies $\omega = \omega_k$ where $\omega_k = E_{\text{col}}(1 + 2k)$ and E_{col} is given by Eq. (29). Harmonics with $k \neq 0$ appear due to the absence of the axial symmetry. Note that $E_{\text{col}} \gg \omega_c$. For low temperatures $T/T_c \ll \sqrt{H/H_{c2}}$, the eigen frequencies are independent of temperature: $\omega_k = E_0(1 + 2k)$. These poles are the collective modes of electrons involved into the vortex motion. For low energies $\epsilon \ll \Delta_\infty \sqrt{H/H_{c2}}$, these modes coincide with multiples of the distance between the true quantum mechanical energy levels determined by Eq. (7). For higher energies, delocalized quasiparticles are involved into the vortex motion.

IV. CONDUCTIVITIES

A. Steady motion

Since γ_O and γ_H do not depend on μ, the integration over $d\mu$ in Eq. (15) can be reduced to the integration over $d\epsilon$. In the sums over n, only the term with $n = 0$ remains because all ω_n with $n \neq 0$ are odd functions of μ. If the Fermi surface has electron-like and hole-like pockets, the conductivities are (compare with Ref.[3])

$$\sigma_O = -\frac{|e|c}{B}\left[N_e \int \langle \gamma_O \rangle_{F,e} \frac{df_0}{d\epsilon}\, d\epsilon + N_h \int \langle \gamma_O \rangle_{F,h} \frac{df_0}{d\epsilon}\, d\epsilon \right],$$

$$\sigma_H = -\frac{ec}{B}\left[N_e \int \langle \gamma_H \rangle_{F,e} \frac{df_0}{d\epsilon}\, d\epsilon - N_h \int \langle \gamma_H \rangle_{F,h} \frac{df_0}{d\epsilon}\, d\epsilon \right].$$

In the moderately clean case, the factors γ_O and γ_H are given by Eq. (21). The conductivities are

$$\sigma_O \sim \frac{Nec}{B}\frac{\Delta_\infty^2 \tau}{E_F} \ln\left(\frac{T_c}{T}\right)\tanh\left(\frac{\Delta_\infty}{2T}\right);$$

$$\frac{\sigma_H}{\sigma_O} \sim \frac{\Delta_\infty^2 \tau}{E_F}\ln(T_c/T) \tag{33}$$

where $N \sim N_e + N_h$. This is similar to the results for an s-wave superconductor[2,3,14].

In the superclean limit $\Omega_0\tau \gg 1$ the factors γ_O and γ_H for low temperatures $T \ll T_c$ are determined by Eq. (25). The general expression for the conductivities are rather complicated. We consider two limits. The universal regime is reached when $\lambda \gg 1$, i.e., when either $\sqrt{H/H_{c2}} \ll 1/(\Omega_0\tau)$ for $T/T_c \ll \sqrt{H/H_{c2}}$ or when $\omega_c\tau \ll T/T_c$ for $T/T_c \gg \sqrt{H/H_{c2}}$. The region of the universal regime is thus much larger than it was predicted in Ref.[1]. This was pointed out by Makhlin[6]. The condition $\lambda \gg 1$ automatically implies that $\omega_c\tau \ll 1$, therefore, both d_1 and d_2 are small. One has from Eq. (25) $\gamma_H = 1 - 2/\pi$, $\gamma_O = 2/\pi$ which results in universal conductivities[1]

$$\sigma_O = \frac{|e|c}{B}(N_e + N_h)\frac{2}{\pi};$$

$$\sigma_H = \frac{ec}{B}(N_e - N_h)\left(1 - \frac{2}{\pi}\right). \tag{34}$$

In the limit $\lambda \ll 1$ we have from Eq. (27)

$$\sigma_O = \frac{ec\,(N_e + N_h)}{B}\,\frac{\Lambda}{\tau};\ \ \sigma_H = \frac{(N_e - N_h)\,ec}{B} \tag{35}$$

where $\Lambda = (N_e\Lambda_e + N_h\Lambda_h)(N_e + N_h)^{-1}$ and

$$\Lambda_{e,h} = \int_0^\infty \langle E_{\text{col}}^{-1}\rangle_{Fe,Fh}\cosh^{-2}\left(\frac{\epsilon}{2T}\right)\frac{d\epsilon}{2T}$$

$$= \left\langle \frac{1}{\sqrt{\Omega_0\omega_c}} + \frac{4T\ln 2}{\pi\Delta_\infty\omega_c} \right\rangle_{Fe,Fh}. \tag{36}$$

Here Ω_0 is determined by Eq. (10) with the logarithmic factor $\ln[(T_c/T)\sqrt{H/H_{c2}}]$. The average is taken over the particle-like and hole-like parts of the Fermi surface. Eq. (35) also agrees with[6]. This regime is reached when either $\sqrt{H/H_{c2}} \gg 1/(\Omega_0\tau)$ for $T/T_c \ll \sqrt{H/H_{c2}}$ or when $\omega_c\tau \gg T/T_c$ for $T/T_c \gg \sqrt{H/H_{c2}}$.

As the temperature approaches T_c, the contribution from fully delocalized states with $\epsilon > \Delta_\infty$ becomes more and more important. The delocalized states give the normal-state Ohmic and Hall conductivities in the limit $T \to T_c$. In the limit $\omega_c\tau \ll 1$, the flux-flow parts of σ_O and σ_H start from the universal values and then first decrease as $\Delta_\infty/T_c \sim \sqrt{1 - T/T_c}$ as long as $\Omega_0\tau$ remains large. The ratio Δ_∞/T_c measures the relative contribution from localized states. With T further approaching T_c, the moderately clean regime is reached, and σ_O becomes proportional to $(\Delta_\infty/T_c)\Omega_0 \propto (1 - T/T_c)^{3/2}$ while σ_H is proportional to $(\Delta_\infty/T_c)\Omega_0^2 \propto (1 - T/T_c)^{5/2}$ being small: $\sigma_H/\sigma_O \sim \Omega_0\tau$ (compare with Ref.[3]). In the limit $\omega_c\tau \gg 1$, the universal regime is not realized, and the conductivity σ_O starts from Eq. (35) and saturates at $\sigma_O \sim (Nec/B)(\omega_c\tau)^{-1}$ while σ_H remains constant.

B. Dispersion of conductivity

1. Low frequency: vortex mass

If both the frequency and relaxation rate are low such that $\lambda, \lambda' \ll 1$ the transverse response is $\langle \gamma_H \rangle = 1$. The transverse force is

$$\mathbf{F}_{\text{env},\perp} = \pi \left(N_e - N_h \right) [\hat{\mathbf{z}} \times \mathbf{u}]. \tag{37}$$

The longitudinal response,

$$\langle \gamma_O \rangle = \frac{4}{\pi E_{\text{col}}} \left(-i\omega + \frac{1}{\tau} \right),$$

gives the force

$$\mathbf{F}_{\text{env},\parallel} = -M \frac{\partial \mathbf{u}}{\partial t} - D\mathbf{u}$$

where D is the friction coefficient and M plays the role of the vortex "mass". Indeed, the force balance takes the from of the Newton's law

$$\mathbf{F}_{\text{L}} + \mathbf{F}_{\text{env},\perp} - D\mathbf{u} = M \frac{\partial \mathbf{u}}{\partial t}. \tag{38}$$

Here the vortex acceleration is a result of action of the Lorentz, transverse, and friction forces. For a steady flux flow without a friction, we would have from Eq. (38) $\mathbf{j}_s = (N_e - N_h) e\mathbf{u}$. In a system with the Galilean invariance, where $N = N_e$ and $N_h = 0$, the sum $\mathbf{F}_{\text{L}} + \mathbf{F}_{\text{env},\perp} = \pi N[(\mathbf{v}_s - \mathbf{u}) \times \hat{\mathbf{z}}]$ is the Magnus force. The friction coefficient and the mass per unit length are

$$D = \pi \left(N_e + N_h \right) \Lambda / \tau; \;\; M = \pi \left(N_e + N_h \right) \Lambda.$$

The effective mass of a d-wave vortex

$$M \sim Nm\xi^2 \left(\sqrt{\frac{H_{c2}}{H}} + \frac{T}{T_c} \frac{H_{c2}}{H} \right)$$

is much larger than the mass of a conventional vortex[15] $M \sim Nm\xi^2$ obtained in the same way. For temperatures $T/T_c \ll \sqrt{H/H_{c2}}$, it is the mass[16] of electrons inside the "vortex core" with the dimensions ξ by $\xi\sqrt{H_{c2}/H}$. The mass increases with temperature. Note that this mass is very large compared with what is usually obtained by calculating the corrections to the kinetic energy of a moving vortex proportional to the second power of its velocity including the electromagnetic energy. For example, the recent result of Ref.[17] is roughly by the factor of $(T_c/E_F)^4$ smaller! We have to stress that this mass appears as a response to a slow acceleration. With an increase in the characteristic frequency of the vortex motion, the response transforms into a highly dissipative resonant behavior where a mass is not an appropriate quantity.

2. High frequency: resonances.

Due to the poles in $\langle \gamma_O \rangle$, the dissipative component of the response has a real part even for $\tau \to \infty$. For $T \ll \Delta_\infty \sqrt{\omega_c/\Omega_0}$, the absorption is concentrated near $\omega_k(p_z) = (2k+1)E_0(p_z)$ with the line width $\delta\omega_k/\omega_k \sim (T/\Delta_\infty)\sqrt{\Omega_0/\omega_c} \ll 1$:

$$\text{Re}\,\sigma_O = \frac{Nec}{B} \sum_k \left\langle \frac{4E_0}{\pi} \delta(\omega - \omega_k) \right\rangle_F.$$

Averaging over the Fermi surface, i.e., integration over dp_z, gives rise to the van Hove singularities in conductivity at frequencies where $\partial E_0/\partial p_z = 0$, i.e., where $\omega = (2k+1)E_0(p_z = 0)$.

For $T \gg \Delta_\infty \sqrt{\omega_c/\Omega_0}$, we have

$$\text{Re}\,\sigma_O = \frac{Nec}{B} \left\langle \frac{\Delta_\infty \omega_c}{T\omega} \sum_{k \geq 0} \cosh^{-2} \left[\frac{\pi \Delta_\infty \omega_c}{4T\omega}(1 + 2k) \right] \right\rangle_F. \tag{39}$$

The conductivity is exponentially small for low frequency $\omega \ll E_g = \pi\Delta_\infty \omega_c/4T$. It starts to rise at the absorption edge $\omega \sim E_g$. The average in the r.h.s. of Eq. (39) is equal to $2/\pi$ for large frequencies $\omega \gg E_g$, when transitions between many resonance modes are excited. This is equivalent to the limit $\lambda \gg 1$ for a steady flow.

The absorption occurs at the collective modes of electrons involved into the vortex motion. For temperatures $T \ll T_c\sqrt{H/H_{c2}}$, these modes are transitions between the true quantum mechanical energy levels determined by Eq. (7). For higher temperatures, delocalized quasiparticles are involved into the vortex motion, and the resonances take place at collective modes with energies of the order of E_g.

V. SUMMARY

For a steady vortex motion, one can identify three regimes in the order of increasing τ: (i) The moderately clean regime with $\Omega_0\tau \ll 1$. In this case, the main response of vortices to the external current is dissipative with only a small Hall angle proportional to $\Omega_0\tau$. The conductivities behave similarly to the s-wave case, see Eq. (33). Next comes the superclean limit $\Omega_0\tau \gg 1$. However, in d-wave superconductors, it further separates into two sub-regimes. The relevant parameter is the energy of collective modes excited by the moving vortex. The vortex dynamics in a d-wave superconductor depends crucially on the behavior of excitations near the gap nodes. Due to the presence of gap nodes, only the excitations with very low energies $\epsilon \ll \Delta_\infty \sqrt{H/H_{c2}}$ are localized in a vortex core. The excitations with higher energies, coming into play for temperatures above $T_c\sqrt{H/H_{c2}}$, are actually the collective

modes where both classically localized and delocalized particles participate. The interplay between the energy of these collective modes $E_{\rm col}$ and the relaxation rate or the frequency of vortex oscillations determines which of the two sub-limits is realized: (ii) Intermediate or "universal" regime with $E_{\rm col}\tau \ll 1$ but $\Omega_0\tau \gg 1$. Here both dissipative and Hall components of conductivity tensor are universal. They are independent of the relaxation time and are only determined by the magnetic field and by the number of electrons and holes under the Fermi surface in the normal state, Eq. (34). (iii) Extremely clean limit $E_{\rm col}\tau \gg 1$ where the dissipative part of the vortex response is small, and the transverse Hall component of conductivity dominates.

Provided the superclean condition $\Omega_0\tau \gg 1$ is satisfied, the universal regime can be realized for temperatures $T/T_c \ll \sqrt{H/H_{c2}}$ under the condition $E_0\tau \ll 1$, i.e., $\Omega_0\tau \ll \sqrt{H_{c2}/H}$. This condition for the universal regime was predicted in Ref.[1]. However, as was noticed in Ref.[6], the universal regime is not restricted to this temperature range. It can also be realized for higher temperatures if $\omega_c\tau \ll T/T_c$. For these temperatures, the dissipative conductivity vanishes only in the extreme clean limit $\omega_c\tau \gg T/T_c$.

The temperature and magnetic-field dependences of conductivities in the extremely clean limit are quite unusual. Extremely clean limit can be reached by increasing the magnetic field above $H/H_{c2} \sim (\Omega_0\tau)^{-2}$ for $T/T_c \ll (\Omega_0\tau)^{-1}$ or above $H/H_{c2} \sim (\Omega_0\tau)^{-1}(T/T_c)$ for temperatures $(\Omega_0\tau)^{-1} \ll T/T_c < 1$. At the crossover from universal to extremely clean regime, the Hall conductivity grows by the factor $\pi/(\pi - 2) \approx 2.75$ while the Ohmic conductivity decreases, Eq. (35). For temperatures $T/T_c \ll \sqrt{H/H_{c2}}$, its field dependence is[1] $\sigma_O \propto H^{-3/2}$; it transforms into[6] $\sigma_O \propto H^{-2}$ for $T/T_c \gg \sqrt{H/H_{c2}}$. The Hall angle approaches $\pi/2$.

At temperatures $T \sim T_c$ but not specifically close to T_c, both Ohmic and Hall conductivities are of the order of Nec/B as long as $\omega_c\tau \ll 1$. They, of course, do not have the universal values any more because all angles contribute similarly to the conductivities. However, the main conclusion is qualitatively the same: dissipation only disappears when $\omega_c\tau \gg 1$. This is the major difference between s-wave and d-wave superconductors: in former, dissipation vanishes already for $\Omega_0\tau \gg 1$. For $T \to T_c$, the superclean condition $\Delta_\infty^2\tau/E_F \gg 1$ fails, and one approaches the moderately clean limit where dissipation dominates.

Having excited an oscillatory motion of vortices by a microwave irradiation at temperatures $T/T_c \ll \sqrt{H/H_{c2}}$ with the frequency $\omega \sim (\Delta_\infty^2/E_F)\sqrt{H/H_{c2}}$, one can observe absorption resonances at the vortex-core states, if the condition $\omega\tau \gg 1$ is satisfied. The required frequency is higher than the cyclotron frequency and has a $\sqrt{H}$ dependence on the magnetic field. For higher temperatures $T/T_c \gg \sqrt{H/H_{c2}}$, one observes an absorption edge at a frequency of the order of $(T_c/T)\omega_c$ where the absorption increases sharply with the increasing frequency due to resonances at the vortex collective modes.

Finally, we find that the response of a vortex to a slow acceleration is characterized by a "mass" term. The mass per unit length, by the order of magnitude, is equal to the mass of electrons inside the vortex core with dimensions ξ by $\xi\sqrt{H_{c2}/H}$ and increases with temperature, i.e., it is enormously larger than the electromagnetic contribution or any other correction to the kinetic energy of a vortex proportional to the second power of its velocity.

APPENDIX A: QUASICLASSICAL SPECTRUM IN D-WAVE SUPERCONDUCTORS

The quasiclassical approximation, a quasiparticle passing near a vortex has a definite trajectory which is actually a straight line characterized by an impact parameter b. This argumentation holds for any symmetry of the order parameter and can be applied to a d-wave superconductor, as well. We solve the quasiclassical Eilenberger equations for the Green functions g, f, and $f^\dagger$ using the same scheme as in Ref.[7]. In the frame with the impact parameter b and the distance along the trajectory s as the coordinates introduced in Section II A, the Eilenberger equations are

$$-iv_\perp\frac{\partial f}{\partial s} - 2\tilde{\epsilon}f + 2\Delta_{\bf p}g = 0 \; ; \tag{A1}$$

$$iv_F\frac{\partial f^\dagger}{\partial s} - 2\tilde{\epsilon}f^\dagger + 2\Delta_{\bf p}^*g = 0 \; . \tag{A2}$$

and $g^2 - f f^\dagger = 1$. Here the functions g, f, and $f^\dagger$ are either retarded or advanced Green's functions. The energy $\tilde{\epsilon} = \epsilon + (2e/c)v_\perp A_s$. For an extreme type II superconductor with the Ginzburg–Landau parameter $\kappa \gg 1$, the magnetic field is nearly homogeneous, $H = n_L\phi_0$ with $A_\phi = H\rho/2$. Since the vector potential component $A_s = -(b/\rho)A_\phi$ we have $\tilde{\epsilon} = \epsilon - eHbv_\perp/2c$.

The order parameter of a d-wave superconductor has the form of Eq. (1) We introduce

$$\eta = -\int_0^s \frac{\partial\phi}{\partial s}\,ds = \frac{\pi}{2} + \alpha - \phi(s). \tag{A3}$$

so that $\cos\eta = b/\rho$ and $\sin\eta = s/\rho$. We put

$$f = f_0 \exp(i\phi + i\eta); \ f^\dagger = f_0^\dagger \exp(-i\phi - i\eta), \quad \text{(A4)}$$

and

$$f_0 = -\theta(s) + i\,\zeta(s); \ f_0^\dagger = \theta(s) + i\,\zeta(s); \quad \text{(A5)}$$

The functions θ and ζ satisfy the equations

$$v_\perp \frac{\partial \zeta}{\partial s} + 2\tilde{\epsilon}\theta - 2i\Delta_{0\mathbf{p}}g\sin\eta = 0; \quad \text{(A6)}$$

$$v_\perp \frac{\partial \theta}{\partial s} - 2\tilde{\epsilon}\zeta - 2i\Delta_{0\mathbf{p}}g\cos\eta = 0. \quad \text{(A7)}$$

where $g^2 = 1 - \zeta^2 - \theta^2$.

For energies close to the eigen energy, i.e., near the poles, the Green functions f and g are large. The function g should be proportional to ζ which has the required symmetry with respect to s. Therefore, we assume that

$$\zeta^2 \gg 1 - \theta^2 \quad \text{(A8)}$$

so that $g = i\sqrt{\zeta^2 + \theta^2 - 1} \approx i\zeta$. The sign is chosen to ensure a decay of g at large distances from the vortex. The function ζ, being the amplitude of the residue of g for the pole, decreases and should completely vanish at large distances.. It is reasonable to assume that, at large distances, $\zeta \to 0$ and $\theta^2 \to 1$. At large distances from the vortex axis, the full Green function

$$f^R = -i\frac{\Delta_\mathbf{p}}{\sqrt{|\Delta_\mathbf{p}|^2 - \tilde{\epsilon}^2}}. \quad \text{(A9)}$$

In combination with Eq. (A5), it gives the required sign of θ. Finally, the boundary condition for the Eilenberger equations are

$$\zeta \to 0; \ \theta \to \pm\text{sign}\left[\sin(2\alpha)\right] \text{ for } s \to \pm\infty. \quad \text{(A10)}$$

The solution is

$$\zeta^{R(A)} = \frac{\text{sign}\left[\sin(2\alpha)\right]v_\perp e^{-K}}{\int_{-\infty}^\infty \left[\tilde{\epsilon} - \Delta_0|\sin(2\alpha)|\cos\eta \pm i\delta\right]e^{-K}\,ds}$$

$$\theta^{R(A)} = 2\text{sign}\left[\sin(2\alpha)\right]$$
$$\times\left(\frac{\int_{-\infty}^s\left[\tilde{\epsilon} - \Delta_0|\sin(2\alpha)|\cos\eta\right]e^{-K}\,ds'}{\int_{-\infty}^\infty\left[\tilde{\epsilon} - \Delta_0|\sin(2\alpha)|\cos\eta \pm i\delta\right]e^{-K}\,ds} - \frac{1}{2}\right)$$

where

$$K = \frac{2}{v_\perp}\int_{s_0}^s \Delta_0|\sin(2\alpha)|\sin\eta\,ds'. \quad \text{(A11)}$$

As a result,

$$g^{R(A)} = \frac{iv_\perp e^{-K}}{C\left[\epsilon - \epsilon_0 \pm i\delta\right]} \quad \text{(A12)}$$

where

$$\epsilon_0(\alpha, b) = C^{-1}|\sin(2\alpha)|\int_{-\infty}^\infty \Delta_0\cos\eta e^{-K}\,ds$$
$$+ eHbv_\perp/2c \quad \text{(A13)}$$

with

$$C = \int_{-\infty}^\infty e^{-K}\,ds. \quad \text{(A14)}$$

Eq. (A13) was obtained in[5,7] for an s-wave superconductor.

For small impact parameters, $b \ll \xi$, one has $|s| = \rho$. Therefore,

$$\epsilon_0(b) = b\left(C^{-1}|\sin(2\alpha)|\int_0^\infty \Delta_0 e^{-K(\rho)}\frac{d\rho}{\rho} + p_\perp\omega_c/2\right). \quad \text{(A15)}$$

where $\omega_c = |e|H/m_\perp c$ is the cyclotron frequency with $m_\perp = p_\perp/v_\perp$. Modulus of charge appears due to the choice of the z axis. Eq. (A15) was obtained in[8]. Due to the Kramer and Pesch effect[7], the core size shrinks at low temperatures to $\xi_1 \sim (T/T_c)\xi$. The spectrum takes the form of Eq. (3) with $L = \ln\left[1/|\sin(2\alpha)|\right]$.

The contribution from the magnetic field increases if the number of vortices increases together with the magnetic field. However, the order-parameter phase gradient $\nabla\chi$ at the position of the vortex core under consideration remains equal to that produced by a single vortex $\nabla_\phi\chi = 1/\rho$ as long as $\rho \ll \rho_{\max}$ where $\rho_{\max}$ is the intervortex distance since contributions from other vortices cancel each other.

Near the gap nodes, where $\sin(2\alpha) \ll 1$, Eq. (A13) for the anomalous branch can be simplified. Significant values of K are determined by $\rho \gg \xi$. Therefore, one can put $\Delta_0 = \Delta_\infty$:

$$K(s) = \frac{2\Delta_\infty|\sin(2\alpha)|}{v_\perp}\int_0^s \frac{s'\,ds'}{\rho'}$$
$$= \frac{2\Delta_\infty|\sin(2\alpha)|}{v_\perp}(\rho - b)$$

The normalization constant is

$$C = 2e^\gamma\int_b^\infty e^{-\gamma\rho/b}\frac{\rho\,d\rho}{\sqrt{\rho^2 - b^2}} = be^\gamma K_1(\gamma) \quad \text{(A16)}$$

where K_1 is the Bessel function of an imaginary argument with $\gamma = 2\Delta_\infty b|\sin(2\alpha)|/v_\perp$. The energy becomes

$$\epsilon_0(b) = \Delta_0|\sin 2\alpha|\frac{K_0(\gamma)}{K_1(\gamma)} + bp_\perp\omega_c/2. \quad \text{(A17)}$$

As a result, for $\xi \ll b \ll \xi/|\sin 2\alpha|$, we get Eq. (3) with $L = \ln[\xi v_\perp/bv_F|\sin(2\alpha)|]$. The low-lying states with energies much below the gap at infinity, $\epsilon \ll \Delta_0|\sin 2\alpha|$ correspond to $b \ll \xi/|\sin 2\alpha|$. For large impact parameters, $b \gg \xi/|\sin 2\alpha|$, we can use the asymptotics of the Bessel functions for large γ and arrive at Eq. (4).

Eq. (A17) holds when Eq. (A8) is fulfilled which is equivalent to

$$\int_0^s (\tilde{\epsilon} - \Delta_0\cos\eta)e^{-K}\,ds \ll v_\perp \quad \text{(A18)}$$

9

for such s that $K \sim 1$. Eq. (A18) is satisfied either for $\gamma \ll 1$ or for $\gamma \gg 1$. In the latter case, the exponent is localized at distances $s \sim \sqrt{b\xi/|\sin(2\alpha)|} \ll b$. Using Eq. (4) we find that the integral in Eq. (A18) is of the order of $v_\perp s/b \sim v_\perp \sqrt{\xi/b|\sin(2\alpha)|} \ll v_\perp$. We see that for $b \gg \xi/|\sin(2\alpha)|$, a particle is localized at $\cos\eta = 1$. i.e., on a line passing through the vortex perpendicular to the momentum direction.

The energy levels for $n \neq 0$ can be found using the full quasiclassical scheme. It works either for $b \gg \xi$ or for $n \gg 1$. We look for a solution for Bogoliubov wave functions in the form $e^{i \int p \, d\rho}$. The quasiclassical momentum is

$$
p = q_\rho \pm \frac{m}{q_\rho} \sqrt{\left(\tilde{\epsilon} + \frac{bv_\perp}{2\rho^2}\right)^2 - \Delta_0^2 \sin^2(2\alpha)} \qquad \text{(A19)}
$$

where $q_\rho = p_\perp \sqrt{1 - b^2/\rho^2}$. The integral along the closed quasiparticle trajectory is

$$
\oint p \, d\rho = 4 \int_b^{\rho_0} \frac{m}{q_\rho} \sqrt{\left(\tilde{\epsilon} + \frac{bv_\perp}{2\rho^2}\right)^2 - \Delta_0^2 \sin^2(2\alpha)} \, d\rho
$$
$$
= 2\pi \left(n + \frac{1}{2}\right) \qquad \text{(A20)}
$$

Consider positive and large $b \gg \xi$ and positive energies $\tilde{\epsilon} \to \Delta_\infty |\sin(2\alpha)|$. At large distances, $1 - \Delta_0/\Delta_\infty \sim \xi^2/\rho^2$. For $b \gg \xi$, the term with b in Eq. (A20) dominates over the large-distance correction to Δ_∞. The zero of the square root in p determines the turning point

$$
\rho_0 = \sqrt{\frac{bv_\perp}{2(\Delta_\infty |\sin(2\alpha)| - \tilde{\epsilon})}}.
$$

For $\rho_0 \gg b$, the integral in Eq. (A20) is logarithmic

$$
\oint p \, d\rho = 4 \sqrt{\frac{b\Delta_\infty |\sin(2\alpha)|}{v_\perp}} \ln \frac{\rho_0}{b}. \qquad \text{(A21)}
$$

Large ρ_0/b correspond to $x_0 \gg 1$ where

$$
x_0 = \pi \left(n + \frac{1}{2}\right) \sqrt{\frac{v_\perp}{b\Delta_\infty |\sin(2\alpha)|}}.
$$

In this limit,

$$
\tilde{\epsilon}_n = \Delta_\infty |\sin(2\alpha)| - \frac{v_\perp}{2b} e^{-x_0}. \qquad \text{(A22)}
$$

Thus, for large n and small enough $b \ll \pi^2 n^2 \xi/|\sin(2\alpha)|$, the energy is exponentially close to the gap $\Delta_\infty |\sin(2\alpha)|$. The derivative with respect to the impact parameter is negative

$$
\frac{\partial \tilde{\epsilon}_n}{\partial b} = -\frac{v_\perp x_0}{4b^2} e^{-x_0}.
$$

The separation between the gap and $\tilde{\epsilon}$ increases with b until $x_0 \sim 1$. The derivative becomes positive for larger impact parameters when $\rho_0 \to b$, i.e., when $x_0 \ll 1$. The energy is

$$
\tilde{\epsilon}_n = \Delta_\infty |\sin(2\alpha)| - \frac{v_\perp}{2b}.
$$

The correction to Δ is two times larger than that obtained for $\epsilon_0(b)$ from the more exact Eq. (4). This is because the full quasiclassical approach does not work well near the turning point. The minimum of energy is reached at $x_0 \sim 1$ and is equal to

$$
\tilde{\epsilon}_n \approx \Delta_\infty |\sin(2\alpha)| \left(1 - \frac{1}{2\pi^2 n^2}\right)
$$

The number of branches with the energy between $\tilde{\epsilon}$ and Δ_∞ is thus given by Eq. (6).

[1] N. B. Kopnin and G. E. Volovik, Phys. Rev. Lett. **79**, 1377 (1997).

[2] N. B. Kopnin and V. E. Kravtsov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 631 (1976) [JETP Lett. **23**, 578 (1976)].

[3] N. B. Kopnin and A. V. Lopatin, Phys. Rev. B **51**, 15291 (1995).

[4] G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **58**, 457 (1993) [JETP Lett. **58**, 469 (1993)].

[5] C. Caroli, P. G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).

[6] Yu. G. Makhlin, Phys. Rev. B **56**, 11 872 (1997).

[7] L. Kramer and W. Pesch, Z. Phys. **269**, 59 (1974).

[8] E. Brun Hansen, Phys. Lett. **27** A, 576 (1968).

[9] M. Franz and Z. Tešanović, cond-mat/9710258.

[10] N. B. Kopnin and G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **64**, 641 (1996) [JETP Lett. **64**, 690 (1996)].

[11] G. E. Volovik, cond-mat/9709159.

[12] Y. Matsuda, N. P. Ong, Y. F. Yan, J. M. Harris, and J. B. Peterson, Phys. Rev. B, **49**, 4380 (1994); J. M. Harris, Y. F. Yan, O. K. Tsui, Y. Matsuda, and N. P. Ong, Phys. Rev. Lett. **73**, 1711 (1994).

[13] M. Stone, Phys, Rev. B **54**, 13 222 (1996).

[14] A. I. Larkin and Yu. N. Ovchinnikov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 210 (1976) [JETP Lett. **23**, 187 (1976)].

[15] N. B. Kopnin, Pis'ma Zh. Eksp. Teor. Phys. **27**, 417 (1978) [JETP Lett. **27**, 390 (1978)]; Physica B **210**, 267 (1995).

[16] G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **65**, 201 (1997) [JETP Lett. **65**, 217 (1997)].

[17] D. M. Gaitonde and T. V. Ramakrishnan, Phys. Rev. B **56**, 11 951 (1997).